QUALIFYING EXAMINATION TOPICS

DEPARTMENT OF MATHEMATICS, DREXEL UNIVERSITY

The qualifying examination is required for all students in the PhD program.

Structure of the examination: four hour closed book test. One-third of the test will be on linear algebra topics; two-thirds on analysis.

Linear Algebra:
The requirements are based on the text: Roger Horn and Charles Johnson, Matrix Analysis. Other sources, listed below, are on permanent reserve (for Math 504) at the library.

B. Noble and J. Daniel, Applied Linear Algebra.
Kenneth Hoffman and Raymond Kunze, Linear Algebra.
Sheldon Axler, Linear Algebra Done Right.
Stephen Friedberg, Linear Algebra.
Gene Golub and Charle Van Loan, Matrix Computations.
L. Trefethen and D. Bau, Numerical Linear Algebra.
Peter Lax, Linear Algebra.
Peter Lancaster, Miron Tismenetsky, The Theory of Matrices.

Topics: A representative list of topics is below. Robert Boyer's study guide will provide more specific advice about preparation for the exam.

- Gaussian elimination, echelon form/row reduced form, LU-decomposition, block matrices and their multiplication, permutation, elementary matrices, rank, linear (in)dependence, determinant, co-factors, Cramer's rule, Cayley Hamilton theorem.
- Vector spaces, subspaces, linear transformations, matrix representation, change of basis, injectivity, surjectivity, inverse.
- Orthogonality, projections, least square approximations and their normal equations, orthogonal bases, Gram-Schmidt orthogonalization, QR factorization, unitary matrices, Schur triangularization.
- Eigenvalues and eigenvectors, diagonalization, Jordan canonical form, spectral theorem for symmetric/normal matrices, Courant-Fischer theorem.
- Norms, condition numbers, Gersgorin disks.
- Positive semidefinite matrices, singular value decomposition, polar decomposition,
- Perron-Frobenius Theorem.

Real Analysis:

The requirements are based on the text: Walter Rudin, Principles of Mathematical Analysis.

Other sources, listed below, are on permanent reserve (for Math 505 and 506) at the library.

Tom Apostol, Mathematical Analysis.
R.P. Boas, A Primer of Real Functions.
R.C. Buck, Advanced Calculus.
H.L. Royden, Real Analysis.
Michael Spivak, Calculus on Manifolds.

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Topics:

- Basic properties of real numbers developed from ordered field and completeness properties.
- Elements of set theory: cardinality, countable and uncountable sets.
- Elements of general metric spaces: convergence, completeness, compactness, connectedness, and their basic properties.
- numerical sequences and their convergence; limitsup and liminf of a sequence.
- numerical series and their convergence: tests for convergence, unconditional and absolute convergence.
- continuous functions and their properties; uniform continuity.
- derivative and its properties; mean value theorem, Taylor expansions.
- Riemann (Stielties) integral and its properties.
- sequences and series of functions; pointwise and uniform convergence; Stone-Weierstrass Theorem; power series
- Calculus of functions of several variables; differentiation, integration and their basic properties: chain rule, inverse and implicit function theorems, change of variables in multiple integrals.
- Lebesgue integral: definition and its properties; fundamental convergence theorems.