

Expertise in Math Doctors

Ian Underwood, Steve Weimar, Robert Rider,
Thomas Hewett, Bruce Char, Jeremy Johnson
and Werner Krandick

Technical Report DU-CS-06-03
Department of Computer Science
Drexel University
Philadelphia, PA 19104
April, 2006

Expertise in Math Doctors

Ian Underwood

Steve Weimar

The MathForum
School of Education
Drexel University

Robert Rider

Thomas Hewett

Department of Psychology and Department of Computer Science
Drexel University

Bruce Char

Jeremy Johnson

Werner Krandick

Department of Computer Science
Drexel University

Technical Report DU-CS-06-03

April 30, 2006

Authors' contact:

Robert Rider

Department of Psychology

Drexel University

Philadelphia, PA 19104

rlr32@drexel.edu

Abstract

Ask Dr. Math is a web-based service for answering mathematics questions from individuals from various ages and backgrounds. Currently, there are over 450,000 Q&A items archived in a database at The MathForum. Although The MathForum is a highly used tool by students, there are currently only a relatively small number of Math Doctors contributing the greatest number of responses, leading to a need for a more effective screening and training process for Math Doctors. This purpose of this study is to evaluate the qualities of expertise illustrated in the responses of the Ask Dr. Math volunteers (Math Doctors). The goal of this evaluation will be to develop a tool for assessing expertise and to outline the specific characteristics of a prototypical “expert” Math Doctor.

Introduction

“Ask Dr. Math” is an online mathematics question and answer service composed of volunteer tutors (Math Doctors) who receive and answer thousands of questions each month. The service began as a forum for students in grades K-12 to have their mathematics questions answered personally by a Swarthmore College math student. Soon, the project was inundated with more questions than the students could answer, and volunteers were recruited from colleges around the country to meet the ever-increasing demand. The service continues to grow in both size and popularity, receiving contributions from over 300 volunteer Math Doctors as well as a number of Internet awards.

Students submit math questions via the “Ask Dr. Math” website and receive e-mail responses. Occasionally, Q & A items that address mathematical principles that often pose problems for students, are moved into a searchable archive, currently organized by grade level (elementary, middle school, high school) and topic (exponents, infinity, polynomials, etc.). In addition, every question submitted to the “Ask Dr. Math” service since 2000 has been saved to an internal archive at The MathForum and, as a result, there is now a vast archive of over 450,000 Q&A items covering an immense breadth of topics and grade levels, offering a unique opportunity for investigating important research questions relevant to mathematics education, cognitive psychology and computer science.

For the purposes of this study, these question and answer items are particularly relevant for understanding the psychology of expertise. Specifically, their content has bearing on the dynamics of “tutoring expertise” in online domains. However this statement requires the qualification that Math Doctors at “Ask Dr. Math” do not typically consider themselves tutors, and by definition, they are not. This issue is addressed in further detail below but, despite the disparity, the large number of inquiries submitted to the service and the diversity of educational levels and backgrounds of those who submit the questions present an opportunity to investigate the dynamics of online, asynchronous, and educationally-based interactions between “novices” and “experts”. *This study will focus on evaluating the qualities of experts in answering math questions in an online mathematics Q&A forum. Specifically, this*

report will focus on (1) presenting the rationale for the development of an expertise assessment tool for online, asynchronous interactions; and (2) outlining the specific components of an expert Math Doctor Prototype model for assessment of expertise in this, and perhaps other, similar, web-based question and answer forums.

Inside “Ask Dr. Math”

When a question is sent to “Ask Dr. Math”, it is automatically assigned an identifying number and sent to a *triage area*, which is essentially a database where it remains available for any Math Doctor to read and answer. The database can be sorted and searched based on a number of criteria (i.e. age). Once a Math Doctor selects a question, he or she uses an email composition tool (the *operating room*) where a variety of tools are at his or her disposal, to compose a response. The tools are continually under development by The MathForum team and are focused on facilitating various strategies for addressing students’ needs, such as quick links to the aforementioned “Ask Dr. Math” archive, unit conversion utilities, responses to frequently asked questions, and a bank of commonly used figures, graphs, symbols and equations.

Once a Math Doctor has completed and sent his or her answer, it moves to a “post-op” area where it remains for an interim period (i.e. as long as the email correspondence continues between the student and the Math Doctor, or until it has been checked for quality by the system administrator). When the correspondence is complete, the string of emails is permanently archived. In some special circumstances, such as the training of a new Math Doctor, the answer is never sent and instead re-enters the triage (see review of current practices of expertise assessment and the tenuring process below) to become available again to the MathDoctor “faculty”.

Current Limitations of the Service

Due to the high volume of questions and the nature of volunteer training and incentive, there exists a large discrepancy between the number of questions received and the number of questions answered, with just 40% of inquiries receiving a response. Also, while some Math Doctors have answered a large number of questions, the majority are involved in a much more limited capacity. In fact, approximately 10% of the current

group of Math Doctors contributes over 90% of answers. While this may be due to several factors, including the time limitations of volunteers and/or some inherent limitation of the service itself, it is apparent that there are too few contributing Math Doctors available to tend to the immense demand for help.

Indeed, many individuals who express interest in becoming a Math Doctor do not continue past the initial screening and training processes. Of those who do complete training and the ensuing period of supervision, few go on to make substantial contributions to the service. It is clear that with only a relatively small number of Math Doctors bearing the majority of the demand for help, a more effective screening and training process is needed. It is clear that the potential for “Ask Dr. Math” to serve as a useful supplement to traditional mathematics education is currently restricted by this uneven contribution. One of the overarching goals of the current project is to improve contribution rates by improving the efficiency of the screening and training processes and by developing a method for increasing the output of existing Math Doctors based on the assessment characteristics of those who contribute with greater regularity.

Background and Development

The task of expertise assessment in online, asynchronous interactions poses several logistical questions for which empirical and theoretical solutions do not currently exist. As will be seen, the expertise assessment model described in this study will be novel in this regard, but based primarily on the existing theoretical model of teaching expertise developed by Sternberg and Horvath (1995) – “The Expert Teaching Prototype” – in conjunction with qualitative observations of the online interactions between Math Doctors and students at “Ask Dr. Math”. As mentioned earlier, it is important to stress that most Math Doctors do not consider themselves “math tutors” per se, and instead describe themselves as expert mathematics consultants who help math students to get quick answers to questions. The development of an assessment tool for evaluating expertise at “Ask Dr. Math,” then will proceed with this notion at its foundation, and modifications to the “Expert Teaching Prototype” will draw on the distinction between traditional teaching and tutoring domains and the unique qualities of an online Q&A forum such as “Ask Dr. Math”.

While there are many differences between the dynamics of the “Ask Dr. Math” service and traditional or even web-based classroom environments, only those pertaining to communication, motivation, orientation, and population will be addressed, as they represent the areas of greatest dissimilarity and will be the most likely factors to impact on the nature of expertise. Below is a more in depth discussion of these factors and selected subjective data from an interview with several current Math Doctors to illustrate these differences.

1. *Communication*: Compared with traditional classroom, where didactic interactions flow naturally from a relatively structured face to face discourse, the communication at “Ask Dr. Math” takes place in something of a vacuum with little, if any, environmental context. Interactions are more condensed, averaging approximately three email messages per problem and asynchronous, taking place over the course of hours, days, or sometimes weeks. Some of the comments from the group interview with practicing Math Doctors revealed some of these limitations. Below are some statements which illustrate this point:
 - a. “It’s really easy to misunderstand one another without gestures”
 - b. “The delay in feedback took adjustment.”
 - c. “A lot of it is educated guess work” [referring to the assessment of student ability and the presenting problem]
 - d. “...the more [the students] are able to communicate, the less likely they have difficulty understanding [the Math Doctor’s response]”
2. *Motivation*: The “faculty” at the Math Forum, as opposed to a traditional or online classroom, is composed of a group of volunteers who offer their time and expertise in mathematics for a variety of reasons (none of which are financial compensation). Indeed, perhaps the most common incentive for these volunteers to contribute to the service is a substantial interest in and curiosity about mathematics coupled with a desire to help others learn. This may translate into more sporadic contribution, and more preferential selection of which questions to answer than would be seen with a paid tutor or teacher, as many Math Doctors report answering questions when time permits rather than during a regular specified time period.

3. *Orientation:* The range of experience of the volunteers at “Ask Dr. Math” differs greatly from that of teachers in more traditional tutoring or teaching positions, which tend to have more stringent regulations for determining who is and is not qualified. For example, some volunteers at “Ask Dr. Math” have real teaching experience whereas others may have worked in advanced mathematics professions and have no formal teaching experience, but instead offer a great deal of practical knowledge and real-world experience. In addition some Math Doctors report preferring to work with children, while others prefer adolescents or adults, and still others simply take the first ten questions in the queue.
4. *Population:* Perhaps one of the most important differences between “Ask Dr. Math” and conventional classrooms or tutoring services is that of the “students” involved. Questions submitted to the “Ask Dr. Math” come from students, teachers, and the general public who have a broad range of experience - from elementary to advanced levels. As opposed to more conventional forms of instruction, where grade level is almost always known, there are typically no indicators of the question-asker’s capability other than his or her age (and any information which can be gleaned from the question). Thus, the volunteer must not only be an expert in mathematics, but must have the ability to determine the aptitude of the individual asking the question with extremely limited information. Math Doctors report that this is typically done by evaluating a section in the question submission form called “thoughts”, where the student presents the Math Doctor with his or her initial attempt at the problem. Several Math Doctors noted that this section was the most informative of the individual’s ability, and that they as consultants were more likely to respond to a question if it was apparent that significant effort was put forth on behalf of the student in the thoughts section.

While teachers/tutors and Math Doctors may seem to have entirely disparate qualities, there are still many bases for comparison, particularly with regard to an appraisal of their relative expertise. The applicability of a similar “expert” model to even the most disparate fields is not new (Ericsson, 1996). This practice stems from an understanding, based on empirical evidence, that the qualities of experts are remarkably

similar across domains. However, the current means for assessing incoming volunteers, potential Math Doctors, does not at present make use of this research.

Assessing Expertise and the Tenuring Process

Currently, prospective Math Doctors must first earn “tenure”, or permission to respond to Math Patients, before they are able to operate independently and without review by the service administrator. The tenuring process currently consists of an initial sample problem - to which the volunteer must provide an original answer - and supervision by the service moderator for an unspecified period of time. After submitting an answer, the volunteer is prompted to re-review their initial response through fourteen on-screen self-checks (see Appendix C). Following further review by the moderator, volunteers are monitored during a training period and their responses to questions in the queue are regularly reviewed for an unspecified period of time. Answers authored by untenured volunteers enter an area known as the *holding tank* until a subjective decision is made by the moderator that the answer has reached an acceptable level of completeness.

Research investigating tutoring efficacy has typically focused on student performance as the primary outcome measure. However, in an environment such as “Ask Dr. Math”, this approach is insufficient, since there are no performance measures for Math Patients. Without indicators of goal completion and virtually no gauge for long-term changes in performance of the Math Patients, tutoring expertise must be assessed via some alternative method. However, despite these limitations and the atypical nature of this informal tutorial service, “Ask Dr. Math” presents a unique opportunity to analyze internet tutoring and expert problem solving with an overwhelmingly robust dataset.

The Expert Teaching Prototype

In the field of expertise research, experts are broadly defined as individuals who have acquired large amounts of knowledge, relevant to a particular task, through training or experience and who have mastered the skills necessary to retrieve and use this knowledge to solve problems quickly, accurately and with relative ease (Ericsson, 1996). Ericsson notes that while the behavioral manifestations of expertise may differ,

research has shown that “the types of acquired mediating mechanisms are remarkably similar across domains.” Thus, expert teachers in traditional classroom settings, and expert Math Doctors should have similar profiles with respect to their ability to draw on their knowledge to assist students in learning mathematics.

In their “Expert Teaching Prototype”, which serves as the basis for the development of the current assessment tool, Sternberg and Horvath (1995) describe three areas in which teachers should be proficient in order to be considered experts: Knowledge, Efficiency, and Insight. These areas are broken down further into sub-areas, with the Knowledge component composed of *content*, *pedagogical* and *practical* knowledge; efficiency divided into *automatization*, *executive control* and the *reinvestment of cognitive resources*; and insight decomposed into *selective encoding*, *selective combination*, and *selective comparison*.

Obtaining *content* knowledge is clearly of primary importance in the development of expertise. It includes the facts and principles relevant to a certain subject area. For Math Doctors, the specific problem and its solution as well as the more general domain of the problem and relevant background knowledge of related domains all fall along the *content* dimension. *Pedagogical knowledge* refers to knowledge of the specific attributes and learning styles of the individual(s) being taught as well as knowledge of how to best teach to those styles and attributes when answering a question. A Math Doctor needs to assess the aptitude of the individual asking the question in order to address the question in the proper terms, defining terms where necessary, using illustrations when appropriate, keying the specific question asked by the patient, using the student’s time effectively and offering further help if needed. The construct of *practical knowledge* refers to the explicit and tacit knowledge of the context in which teaching occurs. A Math Doctor with *practical* knowledge demonstrates a command of teaching methods relevant for this web-based medium, knowing how to maximize the effectiveness of the tools provided to the Math Doctors in combination with their own content knowledge and how and when to reference the archive and other outside sources to aid in addressing the student’s problem.

Sternberg and Horvath divide efficiency into *automatization*, *executive control*, and the *reinvestment of cognitive resources*. In other words, efficiency is concerned

with the ability of teachers to automate routine skills, use time and energy efficiently, solve multiple problems simultaneously and plan, monitor, and evaluate his or her own method of problem solving. Those Math Doctors who contribute readily may be thought of as having automatized the question answering process to the extent that they are able to answer more questions in less time. However, this is a problematic assumption due to the different settings in which the volunteers at “Ask Dr. Math” work (see survey results). *Executive control* is loosely defined here as the ability to anticipate further difficulties and preemptively addresses these potential difficulties in his/her response. The *reinvestment of cognitive resources* is the capacity of the expert to use the time and cognitive resources (saved as a result of their increased efficiency in problem solving) to become creative with their teaching methods. An expert Math Doctor might exhibit *reinvestment of cognitive resources* by including a creative metaphor and a diagram of the problem.

Finally, Sternberg and Horvath refer to insight as involving three aspects: *selective encoding*, or the ability to filter out irrelevant information, *selective combination*, which is the ability to synthesize unrelated information in a discerning manner, and *selective comparison*, or the ability to illuminate the structure and dynamics of an unfamiliar problem using the structure and dynamics of some familiar analogous situation. While insight is certainly an important skill for teachers and tutors to have, attempting to measure it poses many substantial challenges. *Selective Combination* may be conceptualized as the demonstration of the ability to combine seemingly unrelated information in useful ways. This area of “The Expert Teaching Prototype” is more difficult to observe in the brief, single-problem-based interactions at “Ask Dr. Math”. A Math Doctor exercising selective combination may bring together information such as the student’s age and their use of language to inform the language and depth of the answer. *Selective Comparison* is the practice of applying information acquired in one context to a problem encountered in a different context. For a Math Doctor, this type of insight is typically manifest as the use of a relevant analogy that facilitates the explanation of a complicated math concept.

A teacher may have more or less expertise in each of these three areas. To better illustrate this point, consider a hypothetical Math Doctor who is having difficulty

attaining “tenured” status. He may have accumulated a vast *content* knowledge of geometric proofs and principles, but has difficulty addressing questions from younger students. He is capable of writing brilliant answers but he uses language that, while commonplace for him, is unfamiliar to those who have not studied a substantial amount of geometry. The service moderator indicates that he needs to remember that his answers must address underlying math concepts, however he does not exercise the *executive control* needed to realize this is a problem with his use of complicated terminology, so he continues to produce answers that are too complicated for the individual asking the question. As he tries to remediate this problem, he makes use of *selective comparison* by realizing that his work with gardening provides him with a basis for generating relevant analogies that help him convey difficult terms and principles.

In order to assess the relative expertise of Math Doctors more efficiently and systematically, these “expert characteristics” must be well-defined so that they may be employed consistently in a largely objective manner using standardized procedures. It follows that a nomenclature must be built for coding expertise in knowledge, efficiency, and insight. “The Expert Teaching Prototype” will serve as the scaffold for this taxonomy and, hopefully as the means for the development of a more effective expertise assessment tool. A description of this nomenclature and assessment tool are presented in the appendix and accompanied by a set of instructions for implementation.

Appendix A

Assessment of Expertise: Definitions and Method

This appendix describes the interpretation and application of the Sternberg and Horvath “Expert Teaching Prototype” as a scoring system for assessing expertise within the framework of “Ask Dr. Math”. Prior to outlining the analytical method for this project, it is important to define several terms which are used frequently in the ensuing proposal. “Ask Dr. Math” is a prominent component of The MathForum, a web-based service for mathematics education. *Math Doctors* are volunteers from the community who donate their time and expertise in mathematics to answering Math Patient questions. A *Math Patient* refers to any individual who submits a question to the “Ask Dr. Math” service and who is typically experiencing some difficulty with a particular math problem or concept. The term *tenured* refers to the status of Math Doctors who have successfully completed the existing training process outlined above. A *submission* is the initial email sent by a Math Patient email to the “Ask Dr. Math” service. When a Math Doctor responds to a submission, the ensuing interaction between Math Doctor and Patient comprises a *thread*, which is a series of two or more email communications between the Math Doctor and the individual submitting the question. Once an interaction is completed, and the question is resolved, threads become part of the Ask Dr. Math *archive*.

Appendix B

The Expert Math Doctor Prototype: Instructions for implementation and scoring

Each thread will be scored blind to Math Doctor identity and tenure status. This will be accomplished by recruiting an outside party to assign a unique identification code to each Math Doctor. Using the scoring sheet provided, each thread will be assessed on the dimensions listed below and assigned an appropriate score for each item as determined by the rater. The score should reflect the entirety of the thread, and not simply the initial response. The score for each thread is obtained by summing the individual items and dividing by the total possible score.

Knowledge level (Scored 0-6)

Content (0-2): Knowledge of facts and principles relevant to the problem solution and the specific domain of the problem.

0: During the course of the thread, the Math Doctor does not demonstrate an understanding, or demonstrates a misunderstanding of the math concepts related to the subject area in question.

1: During the course of the thread, it is evidenced that the Math Doctor understands the principles and/or facts relevant to the subject area in question by the presence of a reference or mention of those principles and/or facts.

2: The initial response from the Math Doctor demonstrates, by the presence of an original explanation or reference to an explanation of those principles and/or facts, that he or she understands the principles and facts relevant to the subject area in question.

Pedagogical (0-2): Math Doctor demonstrates knowledge of the specific attributes and learning styles of the individual(s) being taught as well as knowledge of how to best teach to those styles and attributes when answering a question.

0: Math Doctor does not demonstrate knowledge of teaching methods, evidenced by a response which is too brief to answer the Math Patient's question or by a response from the Math Patient indicating the response was not sufficient to answer the question. These include messages unsent by Math Doctors-in-training, and those which are sent by tenured Math Doctors which meet the above criteria.

1: Math Doctor demonstrates knowledge of teaching methods and keys the specific question asked by the patient

2: Math Doctor demonstrates knowledge of teaching methods, particularly those relevant for this medium (i.e. referencing the archive, using ASCII figures), and keys the specific question asked by the patient, and uses the patient's time effectively.

Practical (0-2): Math Doctor demonstrates a mastery of Ask Dr. Math procedures (i.e. prompts for Math Doctor).

0: Math Doctor does not demonstrate knowledge of the Ask Dr. Math procedures. This would be evident in a performance of less than or equal to 7/14 items from the existing Math Doctor screening checklist (see below).

1: Math Doctor demonstrates knowledge of the Ask Dr. Math procedures by completing 8-12 items from the Math Doctor screening checklist.

2: Math Doctor demonstrates knowledge of the Ask Dr. Math procedures by completing 13 -14 items from the Math Doctor screening checklist.

Efficiency Level (Scored 0-5)

Automatization: Math Doctor demonstrates an ability to generate a cogent explanation even if confronted with limited information; problem solving strategies are generally automatic thereby leading to a greater number of answers over time.

0: Math Doctor demonstrates a deficiency in the ability to generate a cogent explanation when confronted with limited information, and since problem solving strategies are not automatic, the Math Doctor will produce less than one answer per week.

1: Math Doctor demonstrates some ability to generate a cogent explanation when confronted with limited information, and since problem solving strategies are somewhat automatic, the Math Doctor will produce between one and seven answers per week.

2: Math Doctor demonstrates the ability to generate a cogent explanation when confronted with limited information, and since problem solving strategies are automatic, the Math Doctor will produce between more than seven answers per week.

Executive Control: Math Doctor demonstrates an ability to anticipate further patient difficulties and addresses these potential difficulties in his/her response.

0: Math Doctor demonstrates a lack of ability to anticipate further patient difficulties evidenced by averaging greater than six emails per thread.

1: Math Doctor demonstrates some ability to anticipate further patient difficulties by averaging fewer than six, but greater than three emails per thread and some presence of anticipatory responses to potential Math Patient difficulties.

2: Math Doctor demonstrates an ability to anticipate further patient difficulties by averaging less than three emails per and addresses these potential difficulties in his/her response.

Reinvestment of Cognitive Resources: Math Doctor demonstrates a metacognitive orientation to answering questions, such that with more and more experience (# of threads), responses to patients better approximate the Ask Doctor Math training (sample problem prompts) criteria.

0: Math Doctor performance on the screening checklist does not show a significant increase from baseline over time (i.e. from baseline to the average of the five most recent responses).

1: Math Doctor performance on the screening checklist does show a significant increase from baseline over time (i.e. from baseline to the average of the five most recent responses).

Insight Level (Scored 0-4)

Selective Encoding: Math Doctor demonstrates an ability to distinguish information that is relevant to the problem from information that is not. This measure is based solely on the qualitative analysis of the question and answer threads.

0: Math Doctor does not demonstrate the ability to distinguish information that is relevant to the problem from information that is not.

1: Math Doctor demonstrates improvement in selective encoding with greater experience.

2: Math Doctor demonstrates a mastery of selective encoding.

Selective Combination: Math Doctor demonstrates an ability to combine seemingly unrelated information in useful ways (i.e. age is listed as 25+, but patient is asking how to explain a problem to a 7th grader, so the Math Doctor gives a response that is appropriate for the 7th grader).

0: Math Doctor averages a mismatch between patient age and answer level as measured by the Flesch-Kincaid Grade level.

1: Math Doctor averages a match (+/- one grade) between patient age and answer reading level as measured by the Flesch-Kincaid Grade level.

Selective Comparison: Math Doctor demonstrates the use of a relevant analogy or references some similar problem in the archive.

0: Math Doctor uses fewer than one analogy or archive reference for every ten responses.

1: Math Doctor uses greater than one analogy or archive reference for every ten responses.

Appendix C
Math Doctor Screening Checklist

Did you address the question that was asked? (yes=1; no=0)

Did you start from where the patient is?

Did you address the underlying math concepts?

Would a diagram or graph be helpful?

Did you leave some work for the patient to do?

Is your math correct?

Did you need to explain any notation or terms that you used?

Is your English correct?

Is your formatting neat and readable?

Is your tone friendly and conversational?

Is the length and level of your reply appropriate?

Did you greet the patient by name?

Did you end in a way that invites continued conversation?

Did you have fun? Did it show?

References

- Ericsson, K.A. (Ed.) (1996). *The road to excellence: The acquisition of expert performance in the arts and sciences, sports, and games*. Mahwah, NJ: Erlbaum.
- Sternberg, R., Horvath, J. (1995). A prototype view of expert teaching. *Educational Researcher*, 24(6), 9-17.