Analysis of N-1 Security-Constrained Economic Dispatch using a Linearized Model

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Abstract

The deregulation of the energy sector has created a new market for energy trading. The emergence of this new market allows utility companies to compete in delivering the cheapest power to consumers. However, a true free market cannot be achieved since inherent limits on power transfer are in place. This includes limits on generation, line properties, system stability, and contingency or fault considerations. As the August 2003 blackout demonstrated, an open energy market is more vulnerable to outages as dispatch information is known only minutes or a day before it goes into operation. Security constrained economic dispatch (SCED) aims at delivering the cheapest generation profile given the non-linear constraints listed above. A linearized model is used since it has been shown to be reasonably accurate and deliver much better performance than the full non-linear approach. Two linear programming techniques are commonly used to evaluate these systems namely the dual simplex method (DSM) and primal-dual interior point method (PDIPM). Tradeoffs for both linear programming techniques will be discussed along with an analysis of the computational effort required in each stage of SCED. The objective of this presentation is to examine this simplified model for optimal power flow (OPF), discuss current methods of analysis, provide a timing breakdown of the computational effort required, and discuss a hardware implementation using FPGA (field programmable gate array).

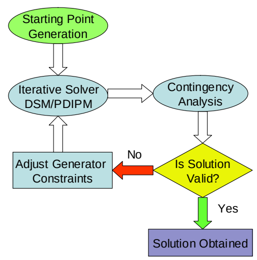
Background/Motivation

Security-constrained economic dispatch (SCED) is a real-time software analysis that is used for generator dispatch and pricing in the energy market. Fast and reliable evaluation of SCED will allow utility companies to compete in a changing energy. Ultimately, this will lead to cheaper energy for the end consumer while maintaining a safe environment for power generation.

Currently, SCED is run mostly on clusters or high-end workstations. However even with large systems, cluster computing delivers only a 4 to 5x performance increase. Field-programmable gate array (FPGA) technology can deliver similar performance at a fraction of the cost. This presentation looks into setting up the SCED problem, comparing the two solvers used in SCED, and provides a performance model for the primal-dual interior point method in FPGA.

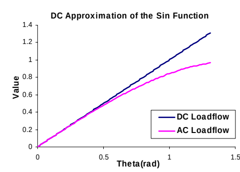
SCED Evaluation Flow Chart

- SCED can be divided into 3 stages
  - Starting Point Generation
  - Base case evaluation
  - Contingency analysis (single line faults)
- The starting point is a initial solution that can be obtained from load flow or through an algorithm
- The Base case is solved using linear programming techniques such as the dual simplex method (DSM) or primal-dual interior point method
- Contingencies are evaluated after to ensure that the dispatch is valid under a single fault condition.



DC Model Assumptions

- All power angles in the system are not greater or less than +/- thirty degrees, respectively. In general, most systems rarely operate at phase angles that exceed these bounds. (A 5% error is incurred at thirty degrees)
- Lossless load flow. This greatly reduces the complexity of the constraint equations. Losses can be assigned after a optimal power flow solution has been obtained.
- Generators are at fixed incremental prices instead of piecewise cost curves or quadratic functions. This is realistic as long as the generators are with a given range of operation.



Case Study Properties

- Four benchmark systems were used
  - The two smaller cases were provided by the University of Washington
  - The two larger systems were provided by PSS/E
- The first chart gives matrix properties used in DC load flow or starting point generation.
- The second gives properties on the matrices used the iterative solver.
- The matrix ADA' represents the constraint matrix associated with the properties of the power system. These constraints are:
  - Line flow constraints
  - Thermal limits
  - Generation Constraints
  - System Balance

System	Branches	NNZ Jacobian	Jacobian Size	Sparsity (%)
118 Bus	186	1,065	181	3.25
298 Bus	410	3,732	526	1.35
1648 Bus	2,602	21,196	2,982	0.24
7917 Bus	13,014	105,522	14,508	0.05

Figure 1: Jacobian matrices used in Load Flow

System	Branches	NNZ (ADA')	NNZ LU (ADA')	Sparsity (%)
118 Bus	186	3827	4566	38.43
298 Bus	410	8321	8524	44.12
1648 Bus	2,602	62,329	81,573	20.75
7917 Bus	13,014	648,863	949,488	13.51

Figure 2: Jacobian matrices used in Load Flow

System Constraints

Objective Function:

$$\min \sum_{i=1}^{N} Ci * Pgi$$

Subject to (ST):

$$\sum_{i=1}^{M} \left(S_{gki} * P_{gi}\right) + T_k = P_k$$

$$T_k^{\min} \le T_k \le T_k^{\max}$$

$$P_{gi}^{\ MIN} \le P_{gi} \le P_{gi}^{\ MAX}, i = 1,2,\ldots N$$

$$\sum_{i=1}^{N} P_{gi} - \sum_{j=1}^{L} P_{lj} - loss = 0$$

The **Objective Function** determines the net cost of power production. We aim at minimizing this value

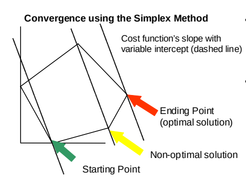
The **"Subject to"** equations describe physical limitations of the system.

1. Line Flow equations determined by line properties.
2. Line MVA thermal limits
3. Production limits or the maximum and/or minimum that can be produced at a generator
4. Balance equations or the net power in each node (should be 0)

Note that all generator dispatches must be > 0

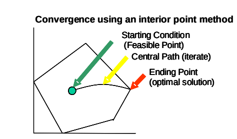
Dual Simplex Method (DSM)

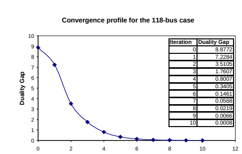
- The DSM solves the problem by *linearly* converging to the optimal vertex.
- The DSM at each iterate determines the variables to add/remove from the basis or optimal set. Thus, the DSM jumps from vertex to vertex until the optimal solution is obtained.
- The algorithm terminates when all adjacent vertices result in a worse solution.
- On smaller systems, the DSM is superior since it initially has no starting point cost and each step is relatively in expensive.
- In the worst case, the DSM's time complexity is exponential, however, this rarely seen in real world problems.



Primal-Dual Interior Point Method (PDIPM)

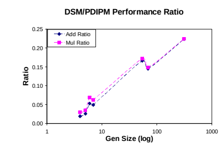
- A Primal-Dual Interior Point Method is the preferred method in most commercial power system packages as it provides a *quadratic* rate of convergence.
- A starting point is chosen from within the feasible set of operating points. This can be taken directly from a load flow program's solution.
- Then the algorithm iteratively converges on the optimal solution.
- The PDIPM uses the gap of the primal and dual solutions to determine if the optimal solution has been obtained.
- This method is preferred when systems are highly sparse and contain 1000s of variables.

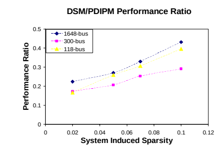




DSM/PDIPM Performance Comparison

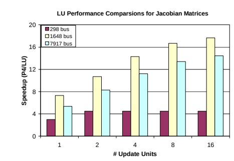
- The figure to the right indicates the ratio of performance between the DSM and PDIPM using a 2% cutoff of the power transmission distribution factors.
- The second figure indicates the dependence of the PDIPM on sparsity and size of the system.
  - As the system size increases the PDIPM performs significantly better.
  - Also as the sparsity increases for a given system, the PDIPM achieves better performance.
- The PDIPM was chosen over the DSM method for the following reasons:
  - Delivers acceptable performance on larger systems with the current model
  - Makes extensive use of linear algebra packages allowing for greater enhancement.
  - Allows the load flow solution to be used as a starting point.

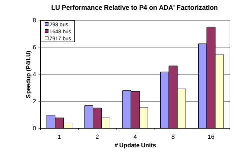




Hardware Model

- A C model was created to simulate the performance gains achievable in using FPGA to perform the lower and upper triangular (LU) matrix decompositions.
- The model assumed the following parameters:
  - Communication is masked by arithmetic operations with a 100% hit rate in the cache.
  - A 200 MHz gate operation utilizing our current floating point unit's (FPU) timing parameters
  - No memory bottleneck. Performs at 200 MHz with 64 bits wide access.
- The two charts represent the relative speedup of the SCED problem in the starting point generation (load flow) and iterative stages.





Results/Discussion

- The starting point (load flow) and iterative solver stages in the SCED are highly dependant on evaluation of linear equations.
- The relative execution times for each process is shown in the top figure. The majority of time is spent in the starting point and iterative stages.
- The net speedup for SCED is shown in the bottom figure. Performance on this case was found to logarithmically dependant on the number of update units in the LU logic.
- The performance of the PDIPM on the largest system indicates that a 5x performance increase can be achieved using FPGA technology.