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Abstract vs Concrete Solutions to Multivariable Truncated Moment Problems

Let $\beta = \beta^{(2d)} = \{\beta_i\}_{i \in \mathbb{Z}_+^n}$, with $|i| \leq 2d$, denote a real n-dimensional multisequence of degree 2d. For example, if n = 2 and d = 1, $\beta^{(2)} = \{\beta_{00}, \beta_{10}, \beta_{01}, \beta_{20}, \beta_{11}, \beta_{02}\}$. For a closed set K in $\mathbb{R}^n$, the truncated K-moment problem asks for conditions on β so that there exists a positive Borel measure μ on $\mathbb{R}^n$, supported in K, such that

$$\beta_i = \int_{\mathbb{R}^n} x^i d\mu(x), |i| \leq 2d.$$

Concrete solutions are known only for a few sets K including $\mathbb{R}$, [0, +∞), [a,b], or when K is a planar curve of degree at most 2. A general solution, proved in collaboration with R.E. Curto, shows that $\beta^{(2d)}$ admits a K-representing measure if and only if the associated Riesz functional ($L_\beta(x^i) = \beta_i$) admits a K-positive extension to polynomials of degree 2d+2. The intricate structure of positive polynomials causes difficulties in establishing K-positivity (so as to apply the extension theorem). We discuss some recent results with Jiawang Nie which resolve the truncated moment problem concretely in some previously open cases, including the case of the bivariate quartic problem (n = d = 2, K = $\mathbb{R}^2$).

Lectures are in Korman Center 245 at 1:00 pm with refreshments preceding the talks at 12:30 pm, also in Korman 245. For additional information contact Greg Naber (Korman Center 255) at gln22@drexel.edu . Directions to Drexel University are available at http://www.drexel.edu/em/directions/directions_uc.html.