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Computational Conformal Geometry and Its Applications

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Riemann surface theory is the intersection field of topology, differential geometry and algebraic geometry, which studies conformal structures of surfaces. Conformal geometry plays an fundamental role in nature and the engineering world.
This talk introduces a series of computational algorithms to compute the major concepts and theorems in conformal geometry and their applications in geometric modeling, computer graphics, computer vision and medical imaging.
According to Riemann uniformization theorem, all metric surfaces can be conformally deformed to three canonical surfaces, the sphere, the plane and the hyperbolic disk. We compute the uniformization metrics using different algorithms: for the spherical case, we apply the heat flow method to compute harmonic maps; for the Euclidean case, we compute holomorphic 1-forms; for the hyperbolic case, we use discrete Ricci flow.
A surface Ricci flow is the process to conformally deform the Riemannian metric such that the Gaussian curvature evolves like heat diffusion on the surface. The method has been applied to 3-manifolds for proving Poincaré conjecture and Thurston geometrization conjecture, which is the generalization of the Riemann uniformization theorem for 3-manifolds. The method to convert continuous Ricci flow to discrete Ricci flow will be discussed in detail.
The conformal geometric algorithms are applied to many engineering applications, including conformal brain mapping, virtual colon flattening, global surface parameterization and general shape matching. Especially, the conformal geometric algorithm is applied to manifold splines. Conventional splines are defined on planar domains. Manifold splines define polar forms directly on manifolds. We show the existence of manifold splines is equivalent to the existence of affine structures on the manifold, which is obstructed by topology. Practical methods to an compute affine atlas for general surfaces and generalize various planar splines to surfaces are explained in details. The results of manifold TSplines, DMS Splines, Powell-Sabin splines will be illustrated.

Lectures are in Korman Center 247 at 1:00 pm with refreshments preceding the talks at 12:30 pm, also in Korman 247. For additional information contact Greg Naber (Korman Center 255) at gln22@drexel.edu. Directions to Drexel University are available at http://www.drexel.edu/em/directions/directions_uc.html.