



Mathematics Department Colloquium

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“Hyperbolic (or Stable) Polynomials and van der Waerden/Schrijver-Valiant Like Conjectures”

December 21, 2006 (Korman 247, 1:00 PM)

The van der Waerden conjecture states that the permanent of an n by n doubly stochastic matrix A satisfies the inequality $\text{Per}(A) \geq n!/n^n$ (VDW bound) and was finally proved (independently) by D. I. Falikman and G. P. Egorychev in 1981. Its worthy successor, the Schrijver-Valiant conjecture on the lower bound on the number of perfect matchings in k -regular bipartite graphs was posed in 1980 and resolved by A. Schrijver in 1998. Schrijver's proof is one of the most remarkable (and highly complicated) results in graph theory. We introduce and prove a vast generalization of the van der Waerden conjecture as well as the Schrijver-Valiant conjecture. Our generalization not only affects the world of permanents, but also has important implications for PDEs, stability and control theory, and complexity theory. Besides, our proof is much shorter and conceptually simpler than the original proofs; it proves the van der Waerden/Schrijver-Valiant conjectures in “one shot”. The main tool in our generalizations and proofs is the concept of hyperbolic polynomials. Hyperbolic polynomials were originally introduced and studied in PDE theory. They are closely related to the multivariate stable polynomials. Van der Waerden/Schrijver-Valiant conjectures correspond to the hyperbolic polynomials being products of linear forms. Our proof is relatively simple and “noncomputational”; it actually improves Schrijver's lower bound, provides a generalization for non-regular and weighted graphs, and uses very basic properties of hyperbolic polynomials (more or less centered around the Gauss-Lukas theorem). The theory is fairly easily generalized to bound from below the number of partial matchings (joint work with S. Friedland). One of the applications results in the best estimate of the 3-dimensional monomer-dimer entropy. Time permitting, I will describe my recent result on analogues of the van der Waerden/Schrijver conjectures for the mixed volume of compact convex sets. This generalization goes beyond hyperbolic polynomials and results in a randomized poly-time algorithm to approximate the mixed volume of n compact convex subsets of \mathbb{R}^n within a multiplicative factor of e^n . This talk is based on the speaker's paper available at <http://xxx.lanl.gov/abs/math.CO/0510452>. See also a shorter version in Proc. of STOC-2006.

Lectures are in Korman Center 247 at 1:00 pm with refreshments preceding the talk at 12:30 pm, also in Korman 247. For additional information contact Greg Naber (Korman Center 255) at gln22@drexel.edu. Directions to Drexel University are available at http://www.drexel.edu/em/directions/directions_uc.html.