Mathematics Department Colloquium
Drexel University
Fall Term 2006-07

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"Linear Matrix Inequality Representation of Convex Sets"

September 18, 2006 (Korman 247, 1:00 PM)

Which closed convex sets C in $R^n$, with an algebraic boundary, admit a representation of the form $C = \{(x_1, \ldots, x_n): A_0+x_1 A_1 + \ldots +x_n A_n \geq 0\}$ for some real symmetric matrices $A_1, \ldots, A_n$? (Here $A \geq 0$ means that the real symmetric matrix A is positive semidefinite.) Such a representation is referred to as a linear matrix inequality (LMI) representation of C. LMIs came to play a central role in systems and control theory in recent years as they admit excellent numerical algorithms and cover a wide range of problems of interest. It turns out that in order to admit an LMI representation the convex set C has to satisfy a strong additional condition (called rigid convexity) which has to do with the algebraic hypersurface a part of which forms the boundary of C. For n = 2 we use the tools of the classical theory of algebraic curves and compact Riemann surfaces to show that, conversely, any rigidly convex set admits an LMI representation. The higher dimensional case remains open. Rigidly convex sets are closely related to so called hyperbolic polynomials, and the LMI representation theorem for n = 2 leads to the proof of a 1958 conjecture of P. Lax on homogeneous hyperbolic polynomials in three variables. If time permits I will also discuss the noncommutative version of the LMI representation problem, where the scalar variables $x_1, \ldots, x_n$ are replaced by matrices of all possible sizes, the boundary of the "noncommutative convex set" being defined by noncommutative algebraic conditions (i.e., polynomials or rational functions in noncommuting variables). This leads to newly emerging "noncommutative real algebraic geometry" and "noncommutative function theory". Surprisingly at first, the noncommutative case seems to be much better behaved than the commutative case. This is very important in applications, since most systems and control theory problems have matrices as natural variables, and lead to noncommutative convex sets of the kind alluded to above. This is joint work with Bill Helton and Scott McCullough.

Lectures are in Korman Center 247 at 1:00 pm with refreshments preceding the talks at 12:30 pm, also in Korman 247. For additional information contact Greg Naber (Korman Center 255) at gln22@drexel.edu . Directions to Drexel University are available at http://www.drexel.edu/em/directions/directions_uc.html.