Mathematics Colloquium

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“Manifold Estimation and Estimation on Manifolds”

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The goal of 3D photography is the inverse of computer aided manufacturing: given a physical object, such as a clay model of a car, a sculpture or a chair, create an electronic representation capturing its shape and appearance. The process typically begins with the acquisition of a cloud of points on or near the surface of the object. This leads to the "manifold estimation problem": given a point cloud in a neighborhood of an (unknown) submanifold of Euclidean space, estimate the submanifold. I will discuss some work on this problem done by our 3D photography group at the University of Washington.

Replacing the point cloud by a probability distribution leads to a natural variational problem: find a submanifold M that is a local minimum for the functional Edist(M), the expected square of the distance to M. One can show that every critical submanifold is a saddle point for this functional. Thus, simple least-squares fitting will not work.

A careful analysis, however, shows that within the class of low-frequency variations, critical manifolds are indeed local minima, and one can estimate the cutoff frequency in terms of the transverse statistics of the probability distribution. Alternately, one can modify E(M) by adding a smoothing term such as harmonic energy: Edist(M) + k Eharm(M). The size of the smoothing parameter k can again be estimated in terms of transverse statistics.